Dynamic Copula Framework for Pairs Trading

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Abstract

Pairs trading is a popular algorithmic trading strategy employed by many practitioners. In recent studies, the Copula Method was proposed to eliminate the rigid assumptions implied by the conventional approaches. However, the existing Copula Method utilizes a static model. On the contrary, it is a stylized fact that stock returns exhibit volatility clustering. Hence, in this paper, a Dynamic Copula framework for pairs trading is proposed using the Dynamic Copula-GARCH model. This aims to further generalize the existing Copula Method. To illustrate the performance of our proposed approach, a comparative analysis, with the conventional method and Copula Method serving as benchmarks, is performed for three Asia Pacific markets (Australia, Japan and Korea). Empirical results show that the proposed approach yields more robust performance as compared to the conventional method and the Copula Method.

Keywords: Pairs Trading; Dynamic; Copulas; GARCH Model
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1 Introduction

Algorithmic trading was popularised in the early 2000s. By 2012, it has already accounted for about 85% of total market volume (Glantz & Kissell, 2013). An algorithmic strategy that presently dominate most markets' order books is pairs trading (Rad et al., 2016). Over the past decade, many researchers have been studying this trading strategy due to its market neutral approach. Gatev et al. (2006) pioneered a comprehensive study of the simplest available pairs trading strategy− the Distance Method (DM). Vidyamurthy (2004) introduced a co-integration technique constructed with a theoretical framework. These two methods are often termed as the conventional approaches for pairs trading. Over the years, the methodologies of pairs trading evolved. There exists a myriad of pairs trading strategies and the discovery of new methods are still emerging. Researchers are constantly trying to improve and propose new strategies to improve the profitability of this popular investment strategy. Xie et al. (2016a) proposed a Copula Method (CM) that frees the strategies from the rigid assumptions undertaken by the former two methodologies. As such, the Copula Method can be viewed as a generalization of the conventional approaches. This makes the Copula Method a more appealing approach in pairs trading as it can be applied in a more general setting. At present, the Copula Method utilizes a static model. However, the importance of dynamic models in equity markets has been illustrated in early works for the field of finance (Erb et al., 1994; Longin & Solnik, 2001; Ang & Chen, 2002). This is further corroborated by the significant increase in the use of dynamic models in finance over the past decade (Cherubini et al., 2012). The main contribution of this paper is the proposal of an alternative approach to pairs trading which encompasses a dynamic dependency structure of stock returns. This approach aims to generalize the Copula Method, allowing pairs trading to yield more robust results in the field of algorithmic trading.

The Distance Method has been a commonly employed strategy in the realm of pairs trading mainly due to two reasons. First, it utilizes a non-parametric model and thus does not subject the stock prices to follow any particular distribution. Second, it follows a straightforward procedure that can be easily implemented. Gatev et al. (2006) were the pioneers of the comprehensive investigation of this simple strategy. They examine it for the U.S. market over a sample period of 40 years from 1962 to 2002. Using committed capital to provide a conservative excess returns figure for their strategy, they document a monthly excess returns of 0.78% the Distance Method's top 5 unrestricted pairs. This indicates the significant profits that can be yield with pairs trading.

However, Do and Faff (2010) have reported a decline in profitability in this simple strategy for the US market, starting from the 1990s. They argue that this decreasing profitability was attributed to arbitrage risk. This is as the simple structure of the Distance Method may have resulted in an increase in arbitrage activities. This in turn decreases the available arbitrage opportunities to be exploited and thus lower profits. In a further study by Do and Faff (2012), they also showed that the distance method, taking into consideration of trading costs, is largely unprofitable after 2002. This serves as a motivation to look into the limitations of the distance method, and devise new strategies that will be able to improve the profitability of pairs trading.

Empirical studies point to the fact that distributions of stock returns are seldom Gaussian. This may be the reason which resulted in the pessimistic view of the Distance Method documented by Do and Faff (2012). The Distance Method uses a single distance measurement which captures the linear dependence between the stock returns. As such, some important non-linear dependency information, like tail dependence, may be unaccounted for.

With a decline in profitability of the Distance Method, there is a need to employ other tools to implement statistical arbitrage trading strategies. The idea of using copulas in pairs trading was first mentioned by Ferreira (2008) in hope to overcome the limitations of the Distance Method. Liew Wu (2013), and Xie et al. (2016a) developed a copula framework on the basis that copula serves as a good candidate to generalize the conventional approaches. The foundation for copula was laid by Sklar's well-known theorem (Sklar, 1959). It provided the link between marginal distributions and their corresponding joint distribution. With copulas, the estimation of the marginal distributions and joint distribution are separated. It frees the normality assumption of joint stock returns implied by the conventional methods. In addition, there exists explicit functions for the copulas, allowing us to better evaluate the dependency between two stocks' returns and thus identify more reliable trading opportunities.

Xie et al. (2016a) proposed the Copula Method and analyse it using 89 utility sector stocks from the U.S. market for a sample period of 10 years (2003 – 2012). Their empirical analysis shows a significantly higher average excess returns for Copula Method with the top 5 and top 20 pair portfolio, as compared to the Distance Method. Furthermore, a lower proportion of trades with negative returns was also observed for the Copula Method. This analysis is further extended to the stocks in the major Asia Pacific market indices, namely SP/ASX 200 (Australia), HSI (Hong Kong), KOSPI 200 (Korea), NIKKEI 225 (Japan) and STI (Singapore) over a sample period of 10 years (2005 – 2014) (Xie et al., 2016b). They document that the Copula Method is generally superior over the Distance Method, except in some cases where they yield similar results. The cases where Copula Method and Distance Method produced similar performances can be seen as instances when Distance Method becomes a special case of Copula Method. These studies, though not as extensive as Gatev et. al's (2006), provide adequate empirical evidence to justify the better performance of the Copula Method, compared to the Distance Method. Furthermore, they follow comprehensive analysis methods employed by Gatev et. al (2006) to analyse the strategies, adding to the robustness of the results obtained.

The Copula Method currently uses a static copula to estimate the joint distribution of the stock pair's returns. This assumes a static dependency structure between the stock returns of a stock pair. However, it is a stylized fact of financial assets that correlations of stock returns between market upturns and downturns differ substantially. Specifically, there is an asymmetric dependence, where two stock returns exhibit a stronger association during a bear market than a bull market. This may be a result of investors' greater uncertainty about the state of the economy (Ribeiro & Veronesi, 2002). Hence, the realized correlation between the two stocks may contain useful information for the prediction of their dependence structure. This suggests that the Copula Method may lack dynamic component in the modelling process. This may have a non-trivial impact on the trades executed by the Copula Method as the mispricing of the stock pair may be incorrectly determined. To address this issue, we introduce the use of dynamic copulas in our proposed strategy.

Dynamic copula is a popular statistical tool used widely in the field of finance (Cherubini et al., 2012). One of the most structured proposals of dynamic copula by Patton (2006), conditional copulas, was motivated by the asymmetry of exchange rate dependence. A significant finding was that time dynamics play an important role in a copula model for the dependence structure of two exchange rates. This spurred a vast amount of literature present employing dynamic copula models in finance. This ranges from modelling financial data (Salvatierra & Patton, 2015; Guegan & Zhang, 2010) to option pricing (Goorbergh et al., 2005). As such, dynamic copula is a potential tool that is able to overcome the limitation of the Copula Method and further generalize the approach for pairs trading.

This paper is related to work over the past decade on pairs trading strategies. We build on the works of Liew & Wu(2013), Xie et al.(2016a) and Xie et al.(2016b), in which the Copula Method is studied, providing the flexibility of modelling the stock returns' dependence structure, capturing important information, like tail dependence, not captured previously. We attempt to overcome the limitations of the Copula Method to further improve the determination of mispricing signals, and subsequently profitability of pairs trading. We propose a dynamic copula framework using Copula-GARCH models in our trading strategy. The proposed Dynamic Copula Method (DCM) will then be compared with the conventional Distance Method and the Copula Method to demonstrate the effectiveness of our strategy.

The remainder of the paper will be as follows. The next section provides a detailed description of our dynamic copula trading methodology. Subsequently, the data selection and analysis methods, as well as the discussion of the empirical results will be provided. Finally, section four concludes the findings of this paper, where future research directions will also be provided.

2 Methodology

In the Dynamic Copula Method, we used a model based on the extension of Sklar's (1959) theorem provided by Patton (2006) as follows: Let $F_{X|W}(.|w)$ be the conditional distribution of $X|W = w$, $F_{Y|W}(.|w)$ be the conditional distribution of $Y|W = w$, $F_{XY|W}(.|w)$ be the joint conditional distribution of $(X,Y)|W = w$, and S be the support of W. Assume that $F_{X|W}(.|w)$ and $F_{Y|W}(.|w)$ are continuous in x and y for all w $\in$ S. Then there exists a unique conditional copula $C(.|w)$ such that $F_{XY|W}(x,y|w) = C(F_{X|W}(x|w), F_{Y|W}(y|w)|w)$, for all (x,y) $\in R^2$, for all w $\in$ S.

There are two main phases in each trading cycle, namely the formation period and the trading period. To better illustrate our proposed framework, we assume that the formation period has a total length of 12 months (252 trading days), while the subsequent 6 months (126 trading days) form the trading period. This gives each trading cycle a total of 378 trading days. Hence, we denote the time series of normalized price of stock i for each trading cycle as $P^t_{X_i}$ for $t = 1, ..., 378$.

2.1 Formation Period

Stock pairs whose prices shows strong co-movement are identified during the formation period. We follow the stock pairs selection process, based on the spread between two stocks, in Gatev et. al (2006).

Let n be the number of stocks to be analyzed. We first form all $\binom{n}{2}$ possible stock pairs. Let X_i and X_j be the stocks for a particular stock pair and denote their respective normalized prices to be $NP^t_{X_i}$ and $NP^t_{Y_i}$. The total sum of spread squares, denoted by $S_{i,j}$, is calculated as in Equation 1.

$$\mathrm{S}_{i,j} = \textstyle\sum_{t=1}^{252}(NP^t_{X_i} - NP^t_{X_j})^2,\ \mathrm{i} \neq j \tag{1}$$

The top five stock pairs with the least $S_{i,j}$ will then be selected to form the pairs trading portfolio for the trading period.

2.2 Dynamic Copula Trading Framework

To capture the dynamicity of the dependence structure, we perform estimation of copula on a rolling basis of one day. For each trading day i ($t = 252 + i$), we define a pseudo formation period to be the period of the previous 252 trading days ($t = i$ to $t = 251 + i$). The pseudo formation period is where necessary parameters are estimated to calculate the mispricing for the next trading day directly after it.

Let P_X^t and P_Y^t be the time series of the stock prices selected in the formation period. The log daily returns series, r_X^t and r_Y^t are calculated as in Equation 2 and 3.

$$\mathrm{r}_X^t = log(\frac{P_X^t}{P_X^{t-1}}), \qquad (i+1) \leq t \leq (251+i) \tag{2}$$

$$\mathrm{r}_Y^t = log(\frac{P_Y^t}{P_Y^{t-1}}), \qquad (i+1) \leq t \leq (251+i) \tag{3}$$

Under the mean reversion assumption for prices of the stock pair chosen, r_X^t and r_Y^t will converge to their respective means in the long run. The means can be estimated by μ_X and μ_Y defined by the means of log returns of Stocks X and Y respectively during the pseudo formation period. As such, given two stocks whose prices co-move during the pairs formation period, the residuals serve as an indication of the relative mispricing between them.

The residuals of the daily log returns time series during the pseudo formation period are then modelled using GARCH(p,q). To illustrate the Dynamic Copula Method in our analysis, we employed the GJR-GARCH(1,1) according to Equations 4 and 5.

$$\mathrm{r}_X^t = \mu_X + \varepsilon_X^t \tag{4}$$

where $\varepsilon_X^t = \sigma_{X,t} Z_t$ with $Z_t = \sqrt{\frac{v_X - 2}{v_X}} T_{v_X}$ and T_{v_X} follows a t-distribution with v_X degrees of freedom; $\sigma_{X,t}^2 = \alpha_X + \beta_X \sigma_{X,t-1}^2 + \gamma(\varepsilon_X^{t-1})^2 + \zeta_X I[\varepsilon_X^{t-1} < 0](\varepsilon_X^{t-1})^2$.

$$\mathrm{r}_Y^t = \mu_Y + \varepsilon_Y^t \tag{5}$$

where $\varepsilon_Y^t = \sigma_{Y,t} Z_t$ with $Z_t = \sqrt{\frac{v_Y - 2}{v_Y}} T_{v_Y}$ and T_{v_Y} follows a t-distribution with v degrees of freedom; $\sigma_{Y,t}^2 = \alpha_Y + \beta_Y \sigma_{Y,t-1}^2 + \gamma(\varepsilon_Y^{t-1})^2 + \zeta_Y I[\varepsilon_Y^{t-1} < 0](\varepsilon_Y^{t-1})^2$.

Assume the respective cumulative distribution functions conditioned on their respective lags are $F_{X|W_X}$ and $F_{Y|W_Y}$, where W_X and W_Y are the lags of Stock X and Y respectively. By Patton(2006), there exists a unique copula linking the conditional marginal distributions of $F_{X|W_X}$ and $F_{Y|W_Y}$. Hence, we can estimate a copula, C, based on the values of $u_t = F_{X|W_X}(e_X^t)$ and $v_t = F_{Y|W_Y}(e_Y^t)$ for $t = i, ..., (251+i)$, where e_X^t and e_Y^t are the realized residuals of stocks X and Y at time t respectively. The copula from the families of Archimedean and elliptical copulas with the highest likelihood is selected.

The optimal estimation of the parameters of the marginals and copula is to use a one-step approach in which all parameters are estimated simultaneously using the maximum likelihood method. However, estimating the marginal and copula parameters jointly is often computationally inefficient. As such, we employ the Inference Function for Margins (IFM)

Method (Joe & Xu, 1996). In this alternative estimation method, the parameters are estimated using a two-step approach. The parameters for the GARCH models for the marginal distributions are first estimated. Conditioned on these estimated marginals' parameters, the copula parameters are then estimated. In both steps, the maximum likelihood method is employed. Due to its higher computational efficiency, the IFM method is often used in copula models for multivariate time-series models (Patton, 2012).

Next, let MI_X^i and MI_Y^i be the mispricings of the two stocks for the i^{th} trading day be defined as the conditional probabilities in equations 6 and 7 respectively.

$$\mathrm{MI}_X^i = P(\varepsilon_X^{252+i} < e_X^{252+i} | \varepsilon_Y^{252+i} = e_Y^{252+i}, W_X, W_Y) \tag{6}$$

$$\mathrm{MI}_Y^i = P(\varepsilon_Y^{252+i} < e_Y^{252+i} | \varepsilon_X^{252+i} = e_X^{252+i}, W_X, W_Y) \tag{7}$$

where e_X^t and e_Y^t are the realized residuals of stocks X and Y at time t, respectively.

The conditional probabilities in Equations 6 and 7 can be calculated using the partial derivatives with respect to v and u respectively as shown in Equations 8 and 9.

$$\mathrm{MI}_X^i = \tfrac{\partial C(u,v|W_X,W_Y)}{\partial v}\Big|_{u=F_{X|W_X}(e_X^{252+i})} \tag{8}$$

$$\mathrm{MI}_Y^i = \tfrac{\partial C(u,v|W_X,W_Y)}{\partial u}\Big|_{v=F_{Y|W_Y}(e_Y^{252+i})} \tag{9}$$

A conditional probability of more than 0.5 indicates that given the price of the partner stock, there is a high chance that the underlying stock should be priced lower than the current price. Hence, in a statistical sense, we can say that stock is relatively over-priced. On the other hand, if the conditional probability is less than 0.5, the stock is viewed as relatively under-priced.

Next, to reflect the mispricing over time and retain the time structure, we consider the mispricing accumulated over a time period. We denote TI_X and TI_Y as the cumulative mispricing of the two stocks. These will act as trading indicators in our trading framework. These indicators take the value 0 before the trading period and upon closing the position of a trade. $(MI_X^i - 0.5)$ and $(MI_Y^i - 0.5)$ are added to TI_X and TI_Y respectively, on a daily basis. We also denote the trading triggers to be D and S. The values of D and S can be obtained via back-testing. Here, we follow Xie et al. (2016a) and set $D = 0.6$ and $S = 2$.

In the event that no trades are open, positions on Stock X and Y will be constructed based on the following cases as shown in Table 1.

[Insert Table 1 here]

Trades will be closed based on the trading indicator used upon opening of trade positions as illustrated in Table 2.

[Insert Table 2 here]

In addition, at the end of the trading period, any opened trades will be closed regardless of the values of the trading indicators.

3 Empirical Analysis

3.1 Data Selection

Our data consists of daily stock data of three Asia-Pacific markets' major indices, namely the S&P/ASX 200 index (Australia), Nikkei 225 index (Japan) and KOSPI 200 index (Korea). The reason for the choice of stocks from these indices is that they form a good representation of their respective markets. The data-set sample period is set at 10 years, from 1 January 2005 to 31 December 2014 and is retrieved from the Bloomberg database. To form our final data-set, stocks with missing data during the sample period, as well as stocks with prices of less than 1 USD are removed. The reason for the removal of such stocks is to closely emulate the practical trading environment, to increase the robustness of the analysis of our proposed framework. The resulting sample consists of 128 stocks from the S&P/ASX 200 index, 204 stocks from the Nikkei 225 index and 169 stocks from the KOSPI 200 index. The analysis is performed on a rolling window basis with a step length of 6 months. This results in a total of 17 trading cycles, each consisting of a formation period and a trading period.

3.2 Analysis Methods

A comparative analysis will be performed to analyse the performance of our proposed Dynamic Copula Method based on several performance indicators in accordance to Gatev et al.(2006). The conventional approach and the Copula Method serve as the benchmarks for the analysis. In a detailed study of pairs trading strategies, the Distance Method and Co-integration Method have shown to yield similar results on a risk-adjusted basis (Rad et al., 2016). As such, the Distance Method will be used to represent the conventional approach due to its relatively simple structure. In addition, we employ the computation of returns based on committed capital, as in Equations 10 and 11.

$$\mathrm{r}_P^t = \frac{\sum_{(X,Y)\in P} w_{(X,Y)}^t r_{(X,Y)}^t}{\sum_{(X,Y)\in P} w_{(X,Y)}^t} \tag{10}$$

$$\mathrm{w}_{(X,Y)}^t = w_{(X,Y)}^{t-1}(1 + r_{(X,Y)}^{t-1}) = (1 + r_{(X,Y)}^{t-1})...(1 + r_{(X,Y)}^1) \tag{11}$$

where $r_{(X,Y)}$ and $w_{(X,Y)}$ denote the returns and weights for each pair within the portfolio respectively.

The daily returns calculated above will then be compounded to obtain monthly returns. In contrast to the computation of fully invested returns, this approach is considered more conservative as it takes into account the amount set aside for potential trades. As such, adopting such a computation will increase the credibility of the results obtained.

3.3 Empirical Results

In order to evaluate the effectiveness of the Dynamic Copula Method, we set the Distance Method and Copula Method as benchmarks and perform a comparative analysis. In order for a trading strategy to be effective, profitability of the approaches has to be analysed. In pairs trading, two factors can affect the profitability, namely the quantity, as well as the quality of trades. These will be discussed in Sections 3.3.1 and 3.3.2 respectively. In addition, we also confirm that the results are robust, using the One Day Wait Strategy and Fama French 3 Factor Model as in Section 3.3.3.

3.3.1 Quantity of Trade

The trading statistics of the three strategies are reported in Table 3. It is observed that trading quantities of both the Copula Method and Dynamic Copula Method are superior to the Distance Method, with the Distance Method generating the least average number of pairs traded per trading period and average number of trades per stock pair. This is consistent with previous literature (Xie et al., 2016a; Xie et al., 2016b). The Dynamic Copula Method has a higher number of trades per pair than the other two strategies, with about 6.65 - 7.08 trades per stock pair compared to 6.39 - 6.75 trades per stock pair for the Copula Method. Hence, the Dynamic Copula Method is more active in generating trades during the sampled period. This implies that the Dynamic Copula Method is able to uncover more trading opportunities. However, this increased number of trades has to come with an accurate detection of the relative mispricing between the stock pair. As such, the quality of the trades will also be examined in Table 4.

[Insert Table 3 here]

3.3.2 Quality of Trade

Table 4 provides the summary statistics of the returns for the three pairs trading strategies for the three markets examined. These include the average excess returns, t-statistic, median, standard error, skewness, kurtosis, minimum, maximum and the percentage of trades with negative excess returns. The Sharpe ratio and Sortino ratio are also reported in the table.

It can be observed that the Dynamic Copula Method yields the highest average excess returns in all three markets examined as compared to the other two strategies (Australia: 1.1343%(DCM) V.S. 0.9418%(CM) and 0.4707236%(DM); Japan: 0.5154%(DCM) V.S. 0.2547%(CM) and -0.0491951%(DM); Korea: 0.5174%(DCM) V.S. -0.162%(CM) and 0.0536231%(DM)). The average excess returns of the Dynamic Copula Method for the Australia market is statistically more significant than the Distance Method. The average excess returns of the Dynamic Copula Method for the Japan market is statistically more significant than both the Copula Method and Distance Method. In addition, the percentage of negative excess returns of the Dynamic Copula Method is also the lowest among the the three strategies in all the markets sampled. This indicates a higher win rate for the Dynamic Copula Method, where it has a higher probability of yielding positive returns when employing the Dynamic Copula Method.

In addition, the Sharpe ratios for the Dynamic Copula Method is generally higher than the other two approaches across all three markets, with the exception of the Australia market. As the Sharpe ratio punishes for 'good' risk, we also consider the Sortino ratio which only considers the downside risk. The Sortino ratios for all three markets is higher for the Dynamic Copula Method, in comparison to the Copula Method and Distance Method. This suggests that the Dynamic Copula Method is able to generate higher returns per unit of risk. Hence, the proposed Dynamic Copula Method is not only able to improve trading opportunities but also the quality of trades.

[Insert Table 4 here]

3.3.3 Robustness Checks

To further enhance our findings, we run a similar analysis using a one-day wait strategy. This further analysis was employed by Gatev et. al(2006) to take into account the effects of a bid-ask spread bounce. Furthermore, trade orders may not be fulfilled once the trade is triggered. These factors may affect the profitability of pairs trading, rendering the need to analyse trades based on a one-day delayed price. The corresponding return characteristics are reported in Table 5. It can be observed that compared to the results obtained without one-day wait in Table 4, the profitability of all three methods decreased as expected, with the exception of the Distance Method in the Korea market. This is a result of slippage in which the entry(exit) trades are not executed at the point when relative pricing of the stocks deviate (converge).

[Insert Table 5 here]

To ensure that the better performance observed for the Dynamic Copula Method is not a result of higher risk, the Fama-French three factor model (Fama and French, 1993) is performed. The results are reported in Table 6. All coefficients for the risk factors are insignificant for the Dynamic Copula Method. This corresponds to the market neutrality of pairs trading strategy. Furthermore, the risk-adjusted returns for the Dynamic Copula Method are significantly positive for the Australia and Japan markets. This indicates that despite the higher returns yield by the Dynamic Copula Method, it does not come with an increase in risk.

[Insert Table 6 here]

We also examine the consistency of the three strategies over time by plotting their respective cumulative returns in Figure 1. The cumulative returns of the Dynamic Copula Method, Copula Method and Distance Method are represented by the green, red and blue plots respectively. It can be observed that generally, the Dynamic Copula Method performed consistently better over the entire 10 year sample period across all three markets sampled.

[Insert Figure 1 here]

Transaction cost plays an important role in evaluating the effectiveness of a trading strategy. Despite the higher average excess returns yield by the Dynamic Copula Method, there is also an increased number of trades executed which implies a higher transaction cost. The transaction fees of major online brokers are examined and we found that the transaction fees varies between 0.1% to 0.2%. For example, a round trip trade costs 0.16% foe the Australia market as quoted from Interactive Brokers. Furthermore, transaction costs can be negotiated with trading size, reducing it to as low as 0.03% per round trip trade. This minimizes the effect of transaction costs on the profitability of pairs trading. Hence, even though this increase in transaction fees of the Dynamic Copula Method will reduce profitability, the results yield is still robust after accounting for transaction costs.

4 Conclusion

Pairs trading has been a popular algorithmic trading strategy employed by many practitioners over the past decades. At present, this market neutral strategy has drawn interest amongst researchers, with many literature investigating and devising new strategies to improve its profitability. The Copula Method is one of many which aims to overcome the limitations of the conventional methods. However, the Copula Method assumes a static structure for both the marginal and joint structure of the stock pair. This is contrary to the stylized facts that financial assets' returns exhibits volatility clustering. Furthermore, the Copula Method also does not take into account dynamic dependence between the stock pair. As such, the model employed by the Copula Method may not accurately reflect the characteristics of the stock pair.

This paper proposes a dynamic copula framework that addresses these downsides by modelling the stocks' returns using Copula-GARCH model as well as a rolling window formation period to account for the dynamic dependency structure. Generally, the Dynamic Copula Method performed relatively better than the Distance Method and Copula Method in terms of average excess returns and risk adjusted returns. The number of trading opportunities has also improved when the Dynamic Copula Method is employed.

Despite the better performance of the Dynamic Copula Method, there still exist a vast amount of areas for further studies. The proposed Dynamic Copula Method currently assumes a GJR-GARCH model with t-innovations for all marginal stock returns due to computational constraints. As such, a further study on modelling the most accurate GARCH model will be able to fully generalize the Copula Method.

Another key area for further studies is the copula-based pair selection. Similar to the Copula Method, the Dynamic Copula Method utilises the distance criteria to determine the stock pairs to be traded. Krauss and Stubinger (2015) mention that the choice of the top stock pairs with minimum squared distances introduces a selection bias. This subjects the Dynamic Copula Method to the same question as the Copula Method – Is the pairs selection process able to accurately select 'good' stock pairs in terms of the Dynamic Copula Method algorithm? As such, further studies on a copula-based selection criteria should be investigated. The aforementioned provide interesting topics for future research which we hope will help enhance our proposed framework.

Open Trade Triggers		
Positions	Long	Short
TI_X reaches D/ TI_Y reaches $-D$	X	Y
TI_X reaches $-D$/ TI_Y reaches D	Y	X

Table 1: Open Trade Trigger for Dynamic Copula Pairs Trading Strategy

Close Trade Triggers	
Open Trade Trigger	Close Trade Trigger
TI_X reaches D	$TI_X \leq 0$ or $TI_X \geq S$
TI_Y reaches $-D$	$TI_Y \geq 0$ or $TI_Y \leq -S$
TI_X reaches $-D$	$TI_X \geq 0$ or $TI_X \leq -S$
TI_Y reaches D	$TI_Y \leq 0$ or $TI_Y \geq S$

Table 2: Close Trade Trigger for Dynamic Copula Pairs Trading Strategy

Trading Statistics				
		DM	CM	DCM
Australia	Average No. of Pairs Traded Per Trading Period	4.5194	5	5
	Average No. of Trades Per Pair	1.8824	6.4824	6.6353
	Std. Dev. of No. of Round Trips Per Pair	1.34	2.4814	2.1036
	Average Time Pairs are Open (months)	1.9445	4.5664	4.5882
	Std. Dev. of Time Open Per Pair (months)	1.2538	0.5078	0.4297
Japan	Average No. of Pairs Traded Per Trading Period	4.8235	5	5
	Average No. of Trades Per Pair	1.9059	6.3882	6.8471
	Std. Dev. of No. of Round Trips Per Pair	1.2014	2.2044	1.5392
	Average Time Pairs are Open (months)	1.9725	4.4275	4.3972
	Std. Dev. of Time Open Per Pair (months)	1.3224	0.4677	0.4621
Korea	Average No. of Pairs Traded Per Trading Period	4.9412	5	5
	Average No. of Trades Per Pair	1.9412	6.7529	7.0824
	Std. Dev. of No. of Round Trips Per Pair	1.1987	2.1095	1.7942
	Average Time Pairs are Open (months)	1.7115	4.5815	4.544
	Std. Dev. of Time Open Per Pair (months)	1.2555	0.4262	0.4093

Table 3: Trading Statistics of Pairs Trading Strategies

Profitability Statistics				
		DM	CM	DCM
Australia	Average Excess Returns	0.004707*	0.009418***	0.011343***
	Newey-West t-Statistic	1.9472	2.9935	2.8166
	Sharpe Ratio	0.1476	0.2609	0.2504
	Sortino Ratio	0.3207	0.5823	0.6998
	Median	0.00008711	0.005341	0.007645
	Standard Error	0.02441	0.03178	0.04067
	Skewness	1.9632	1.134	2.8085
	Kurtosis	10.7802	5.2661	15.9482
	Minimum	-0.04764	-0.07178	-0.06265
	Maximum	0.1358	0.1137	0.2527
	% of Excess Return < 0	47.0588	44.1177	40.1961
Japan	Average Excess Returns	-0.000492	0.002547	0.005154*
	Newey-West t-Statistic	-0.2674	0.8539	1.8706
	Sharpe Ratio	-0.08564	0.0483	0.1454
	Sortino Ratio	-0.1561	0.09245	0.2932
	Median	-0.0007261	0.0000134	0.005201
	Standard Error	0.01858	0.03012	0.02782
	Skewness	1.9983	0.9276	0.3921
	Kurtosis	13.4546	5.8553	3.1823
	Minimum	-0.03977	-0.06125	-0.04995
	Maximum	0.1079	0.1367	0.08457
	% of Excess Return < 0	50.9804	50	44.1177
Korea	Average Excess Returns	0.0005362	-0.00162	0.005174
	Newey-West t-Statistic	0.1801	-0.3925	1.1961
	Sharpe Ratio	-0.01855	-0.0649	0.09333
	Sortino Ratio	-0.03252	-0.105	0.1936
	Median	-0.003382	-0.00038	0.002267
	Standard Error	0.03007	0.0416	0.04369
	Skewness	0.4366	0.1963	0.6058
	Kurtosis	4.22	3.4271	3.5794
	Minimum	-0.09545	-0.111	-0.08196
	Maximum	0.08362	0.1166	0.1539
	% of Excess Return < 0	55.8824	50	49.0196
*Note: *, **, *** represent 10%, 5% and 1% significance levels respectively*				

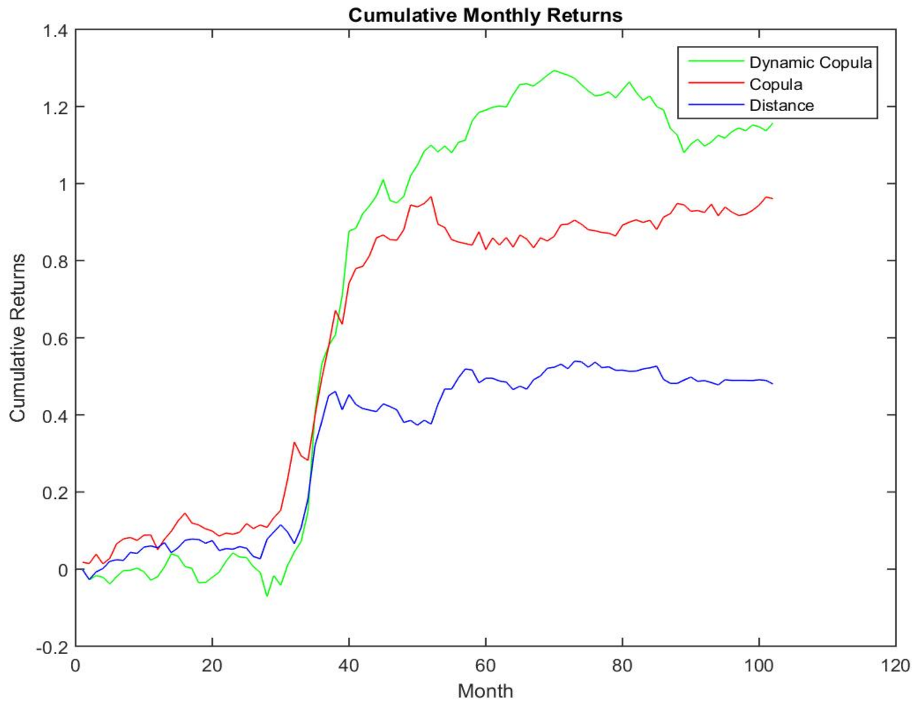
Table 4: Returns Characteristics of Pairs Trading Strategies

Profitability Statistics (One Day Wait Strategy)				
		DM	CM	DCM
Australia	Average Excess Returns	0.003265	0.008024**	0.00977**
	Newey-West t-Statistic	1.438	2.5351	2.5492
	Median	0.0006166	0.004297	0.007104
	Standard Error	0.02293	0.03197	0.03871
	Skewness	2.4697	1.3297	1.5435
	Kurtosis	18.01808	5.5631	11.00186
	Minimum	-0.05748	-0.04802	-0.1262
	Maximum	0.1496	0.1168	0.2011
	% of Excess Return < 0	45.09804	43.1373	38.2353
Japan	Average Excess Returns	-0.001055	0.0016	0.001132
	Newey-West t-Statistic	-0.6120	0.5953	0.4152
	Median	-0.0009007	0.001279	0.000696
	Standard Error	0.01741	0.02714	0.02754
	Skewness	1.6968	0.7227	0.3232
	Kurtosis	13.4173	4.4452	3.7696
	Minimum	-0.0427	-0.0521	-0.0691
	Maximum	0.1006	0.1105	0.08635
	% of Excess Return < 0	52.9412	48.03922	50
Korea	Average Excess Returns	0.001747	-0.00286	0.002426
	Newey-West t-Statistic	0.6341	-0.6631	0.5783
	Median	-0.0005904	0.002807	0.000448
	Standard Error	0.02782	0.0436	0.04236
	Skewness	0.5543	0.3062	-0.02789
	Kurtosis	4.5016	3.6024	4.0929
	Minimum	-0.07624	-0.1072	-0.1392
	Maximum	0.09662	0.136	0.1154
	% of Excess Return < 0	51.9608	49.01961	50
*Note: *, **, *** represent 10%, 5% and 1% significance levels respectively*				

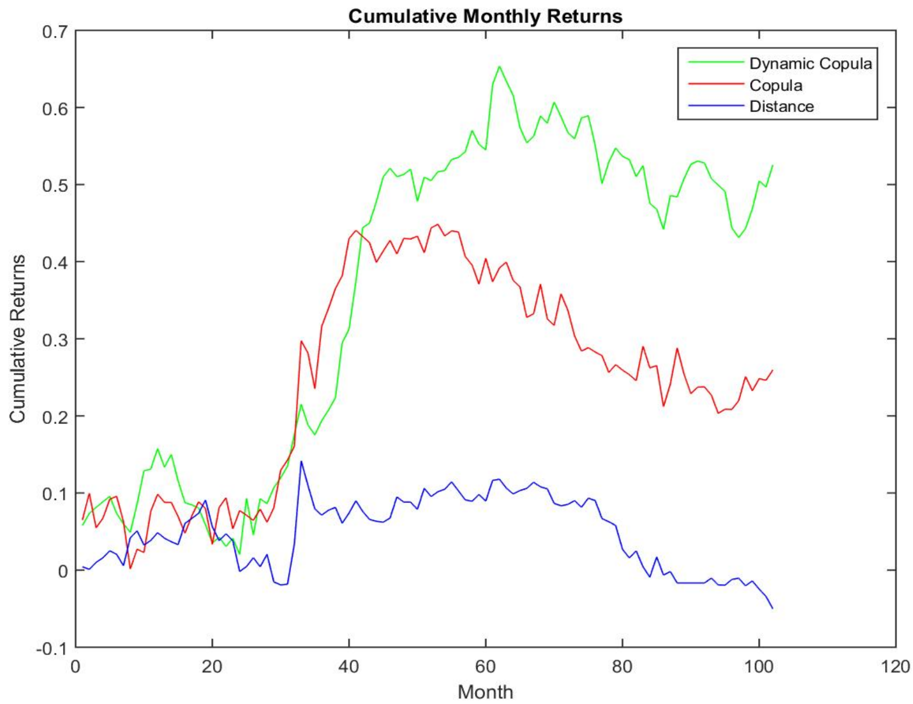
Table 5: Returns Characteristics of Pairs Trading Strategies (One Day Wait)

Fama French 3 Factor Model							
		DM		CM		DCM	
		Coefficient	t-statistic	Coefficient	t-statistic	Coefficient	t-statistic
Australia	Alpha	0.4959**	2.0752	0.9327***	2.8671	1.1302***	2.6955
	Mkt $-R_f$	-0.08034	-2.1527	-0.00996	-0.196	-0.00803	-0.1227
	SMB	-0.0361	-0.4234	0.1175	1.0126	-0.06381	-0.4266
	HML	0.1537	1.5417	0.114	0.8399	0.01068	0.06107
Japan	Alpha	-0.05894	-0.3126	0.2058	0.6797	0.4961*	1.7622
	Mkt $-R_f$	-0.0218	-0.5014	-0.05285	-0.7572	0.06996	1.07794
	SMB	-0.01995	-0.2471	0.1091	0.8414	0.0139	0.1153
	HML	0.03704	0.417	0.1515	1.0627	0.04514	0.3405
Korea	Alpha	0.01754	0.05728	0.03676	0.08977	0.4688	1.04541
	Mkt $-R_f$	0.04871	1.01863	-0.08882	-1.3889	0.06028	0.8608
	SMB	0.1023	0.9369	-0.1897	-1.2983	-0.08405	-0.5254
	HML	0.01176	0.09206	-0.4627***	-2.7091	-0.0664	-0.35505
*Note: *, **, *** represent 10%, 5% and 1% significance levels respectively*							

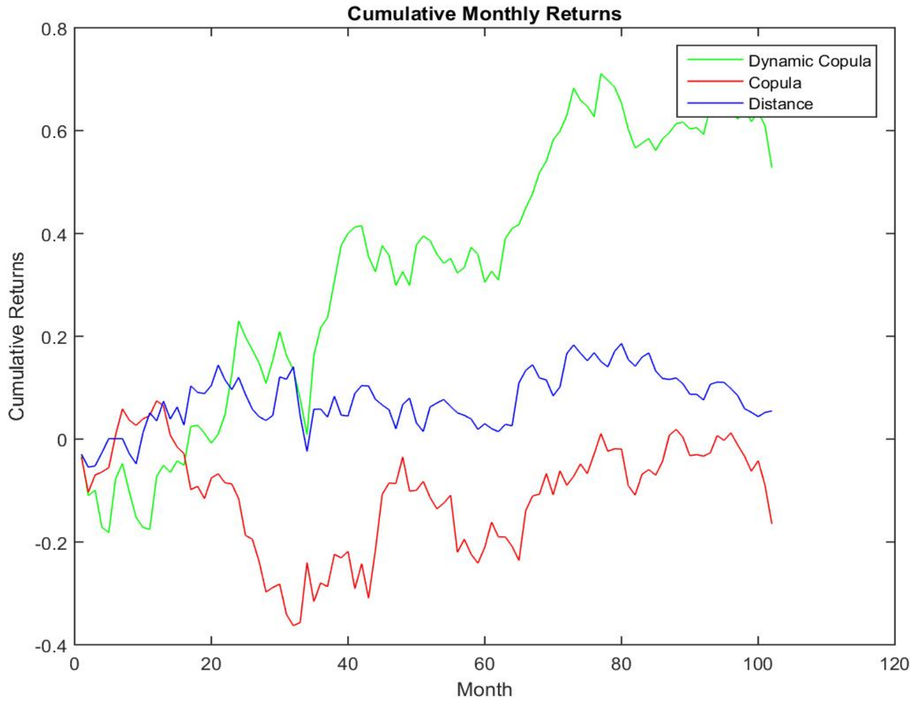
Table 6: Risk-Adjusted Returns of Pairs Trading



(a) Australia: S&P/ASX 200



(b) Japan: NIKKEI 225



(c) Korea: KOSPI 200

Figure 1: Cumulative Returns Plot for Pairs Trading

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